BBB-003-001105 Seat No. ________

B. Sc. (Sem. I) (CBCS) Examination

July - 2021

Mathematics : M - 101

(Geometry & Calculus)

(Old Course)

Faculty Code : 003

Subject Code : 001105

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) All questions of SECTION-A carry equal marks and each question of SECTION-B carry 25 marks.

(3) Write answer of each question in your main answer sheet.

SECTION-A

1 Answer the following questions in short : **20**

(1) Convert the polar co-ordinate $\left(1, \frac{\pi}{2}\right)$ into cartesian form.

(2) Convert the cartesian co-ordiante (1, 1) into polar form.

(3) Convert the cartesian equation $x^2 - y^2 = a^2$ into polar equation.

(4) State the equation of straight line in $p-\alpha$ form.

(5) Write Relation between cartesian co-ordiante and cylindrical co-ordinate.

(6) If $y=\sin(ax+b)$ then write the formula of y_n.

(7) If $y=x^7$ then $y_7 =$ __________ .

(8) If $y=x^{10}$ then y_{11} = __________.

(9) Define : Strictly Increasing function.

(10) Evaluate $\lim\limits_{x\to 1} \dfrac{\log x}{x-1}$.

(11) $\lim\limits_{x\to 0} \dfrac{a^x - 1}{b^x - 1}$ = __________

(12) Write the general solution of $y = xp - p^2 + \log p$.

(13) State the necessary and sufficient condition for the differential equation $M\,dx + N\,dy = 0$ to be exact.

(14) State Bernoulli's differential equation.

(15) Write the general solution of the equation

$$\left(D^2 - 3D - 4\right)y = 0.$$

(16) Find $\frac{1}{D}x^2$.

(17) $\frac{1}{D^2+7}\sin 2x =$ __________.

(18) $\int_0^{\pi/2} \sin^4 x\, dx =$ __________.

(19) $\int_0^{\pi/6} \cos^6 3x\, dx =$ __________.

(20) Write the value of $\int_0^{\pi/2} \sin^m x \cdot \cos^n x dx$ when $n = 1$.

SECTION-B

2 (a) Answer any **three** : **6**

(1) Find the distance between two polar co-ordinate $A\left(a, \frac{\pi}{2}\right), B\left(3a, \frac{\pi}{6}\right)$.

(2) Obtain the centre and radius of the given circle $r^2 - 8r\cos\left(\theta - \frac{\pi}{6}\right) + 12 = 0$.

(3) Change polar equation $r\cos\theta = 2\sin^2\frac{\theta}{2}$ into cartesian equation.

(4) If $y = e^{5x} \sin 3x$ then find y_n.

(5) Show that the function $2 - 3x + 6x^2 - 4x^3$ is strictly decreasing in every interval.

(6) Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$.

(b) Answer any **three** : **9**

(1) Obtain the equation of straight line passing through the point $(1, \pi)$ and $\left(2, \frac{\pi}{2}\right)$.

(2) Find the sphere for which $A(2, -3, 4)$ and $B(-2, 3, -4)$ are the extremities of a diameter.

(3) Find nth derivative of e^{ax}.

(4) If $y = \left(\sin^{-1} x\right)^2$ then show that

$\left(1 - x^2\right) y_2 - x y_1 - 2 = 0$.

(5) Verify Rolle's theorem for the function

$f(x) = x^2 - 2x, \forall x \in [-1, 3]$.

(6) Find $\lim_{x \to 0} \frac{x \cos x - \log(1 + x)}{x^2}$.

(c) Answer any two : **10**

(1) Find the equation of the sphere which touches the sphere $x^2+y^2+z^2-x+3y+2z-3=0$ at the point $(1,1,-1)$ and passes through the origin.

(2) State and prove Liebnitz's theorem.

(3) State and prove Lagrange's mean value theorem.

(4) For $f(x)=x^x, g(x)=x\log x$ where $a<c<b$ show that $b^b-a^a=c^c[b\log b-a\log a]$.

(5) Expand $\sqrt{x}$ in ascending powers of $(x-4)$.

3 (a) Answer any three : **6**

(1) Solve the differential equation $y=px+\frac{m}{p}$.

(2) Show that the equation $(x^2-ay)dx+(y^2-ax)dy=0$ to be exact.

(3) Solve : $p^2-7p+10=0$.

(4) Find : $\int \cos^5 x \, dx$.

(5) Solve : $\cos^2 x \frac{dy}{dx} + y = \tan x$.

(6) Prove $\int_0^{\pi/2} \sin^6 x \cdot \cos^8 x \, dx = \frac{5\pi}{4096}$

(b) Answer any three : **9**

(1) Solve : $y = 2px - \frac{1}{3}p^2$

(2) Solve : $y - 2px = \tan^{-1}\left(xp^2\right)$

(3) Solve : $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$.

(4) Solve : $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = e^{5x}$.

(5) Solve : $\left(D^2 + 4\right)y = \sin 2x$.

(6) Prove that $\int_0^2 \frac{x^4 dx}{\sqrt{4 - x^2}} = 3\pi$.

(c) Answer any two : **10**

(1) Find the solution of Bernoulli's differential equation.

(2) Derive the formula to solve linear differential equation of first order and first degree.

(3) Prove that $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}, f(a) \neq 0$.

(4) Obtain the reduction formula for $\int \cos^n x dx, n \in N$.

(5) Obtain reduction formula for $\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx$;

$m, n \in N$.